

# Simulation Methods in Epidemiologic Research and Learning

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# Random Error and 95% CIs

- **If you ask most people, a 95% confidence interval from 1.1 to 2.3 means:**
  - There is a 95% chance that the true value is between 1.1. and 2.3
  - This is not correct
- **If statistical model is correct and no bias, a confidence interval derived from a valid test statistic will, over unlimited repetitions of the study, contain the true parameter with a frequency no less than its confidence level (e.g. 95%)**
  - Simple simulation helps make the distinction

# Simulate the height of 1000 people with a mean of 65 and std of 5

```
data master;
  do j = 1 to 1000;
    seed1=-1;
    x= rannor(seed1);
    height = 65+5*x;
    output;
  end;
  drop j seed1;
run;
```

```
proc means data=master n mean std min max; var height;
  title "True population mean height";
run;
```

```
*****;
* REPEATED SAMPLING **;
*****;
```

```
%macro rep();
  %do j = 1 %to 1000;
    data onerep;
      retain seed1;
      * initialize the seed variable;
      if seed1 eq . then seed1 = -1;
      * loop until designated number of controls are found;
      do j = 1 to 20;
        * choose a random person in the database;
        lookat = round(1000*ranuni(seed1),1)+1;
        * hold that record for the new dataset;
        set master point=lookat;
        * output the record to the new dataset if they are eligible;
        output;
      end;
      drop j seed1;
    stop;
  run;

  proc means data=onerep mean lclm uclm noprint;
    var height; output out=outset lclm=lclm uclm=uclm;
  run;
  data outset; set outset;
    if lclm le 65 le uclm then included=1; else included=0;
    attrib included label="Did the 95% CI include the true value?" format=yn.;
  run;

  proc append base=newset data=outset force; run;
%end;
```

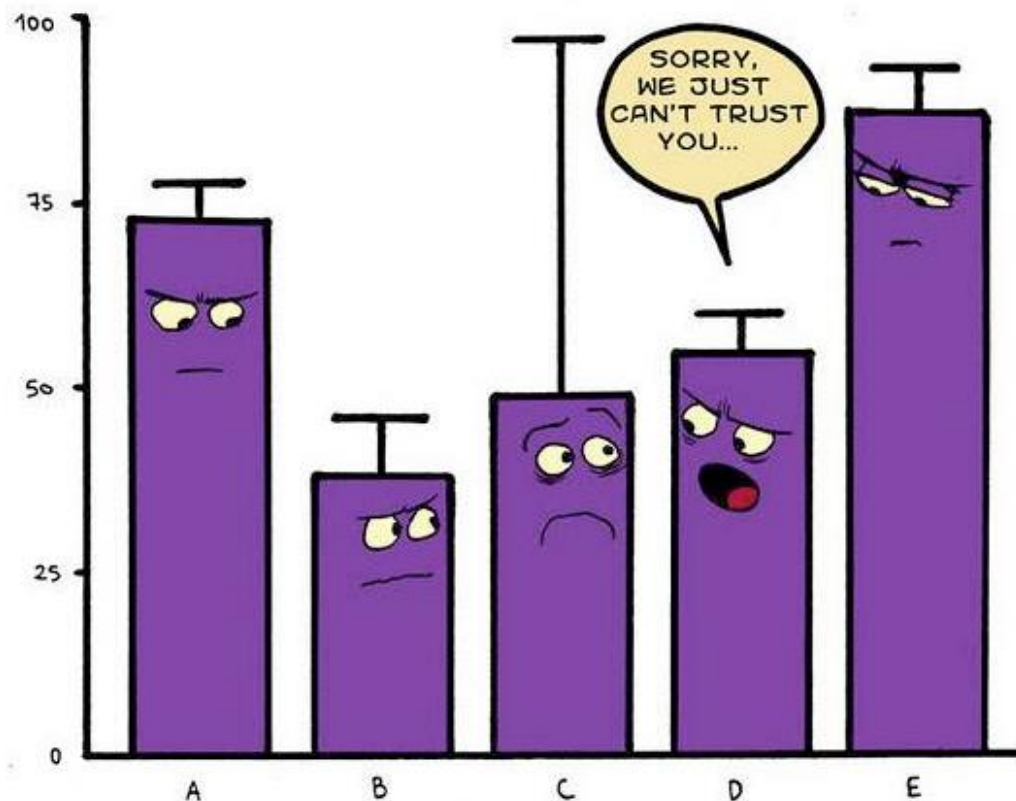
From the initial 1000, simulate 1000 datasets each drawn from the original of size 20 and for each calculate a mean and 95% CI

# How Often Did CI Contain the Truth?

	N	Mean	Std Dev	Minimum	Maximum
Full sample	1000	65.3225048	4.9252091	50.7579163	86.5469094

Did the 95% CI include the true value?

included	Frequency	Percent	Cumulative Frequency	Cumulative Percent
No	53	5.30	53	5.30
Yes	947	94.70	1000	100.00



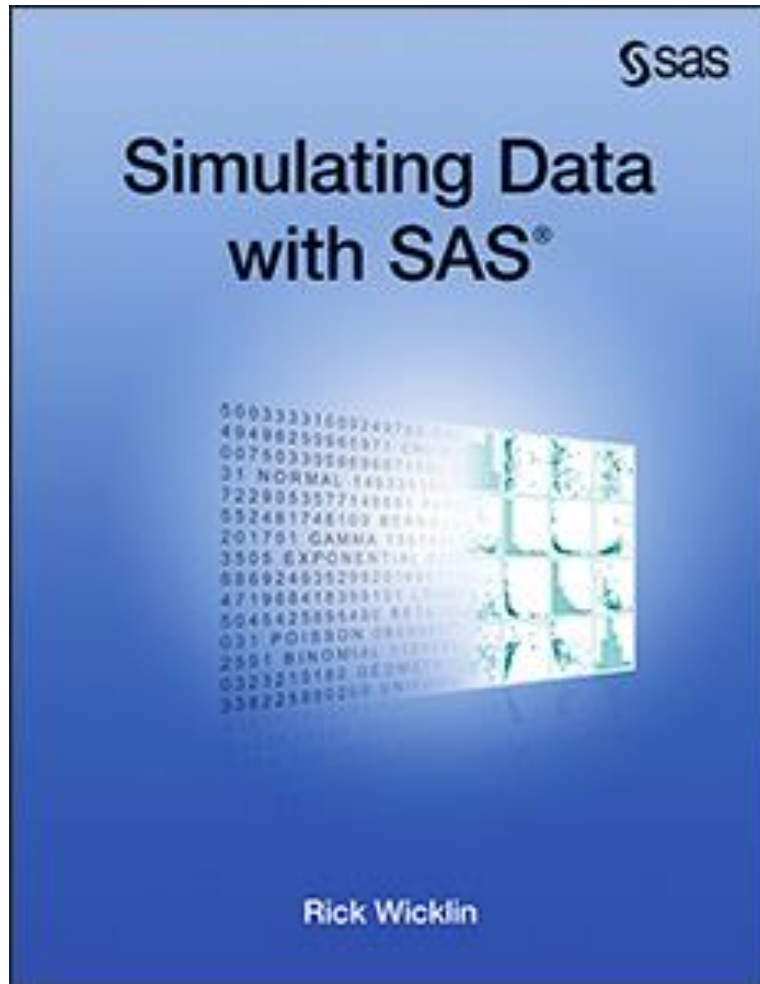
# Outline

- **How SimPLE started**
- **What we've done**
- **How you can do it**
- **Some examples**
- **Why it is important**

# **DISCLAIMER:**

**I am not an expert in data  
simulations ...  
and this is the point!**

# A Useful SAS Book



# Motivation

- **In my doctoral program I was always wanting a “confounded” dataset when TAing or getting ready for exams, yet at first I didn’t know how to create one**
  - Found out that in order to simulate it, you have to understand it well enough
  - Started to realize what I didn’t know
  - Started to realize I could figure out things myself
- **I had a colleague who said that he took a class in which for every concept they learned, they had to simulate a dataset that illustrated that problem**



# Epi Doctoral Qualifier Question

**Below is a shell table for a dataset on the relationship between an exposure E and an outcome D stratified by a covariate C. Assume that we could know each person in the study's counterfactual susceptibility type (Type 1-4)\*. Create a dataset with the following properties and fill in the table below:**

1. The crude E-D relationship is confounded by C (by statistical criteria)
2. The C stratum-specific estimates of the E-D relationship are unconfounded (by statistical criteria)
3.  $P1$  is not equal to  $Q1$ \*
4. There is no effect measure modification by C of the ED relationship on the difference scale but there is effect measure modification on the relative scale



# So Was the Birth of SimPLE

- **SIMulating Problems for Learning Epidemiology**
- **Goals:**
  - Bring together doctoral students from epidemiology and environmental health to learn
  - Everyone contributes
  - We are all beginners
  - We all choose a topic to try to understand better
- **Took us a few sessions to cover some very simple concepts and everyone was off and running**
  - Message: basic simulation for learning is not hard to do!

# What Have We Covered

- **Simulating datasets**
- **Simulating datasets with particular structures**
  - **Confounding**, collider bias, effect measure modification
- **Simulating dataset from the main dataset with bias**
  - Selection bias, measurement error
- **Understanding M bias**
- **Quantitative bias analysis**
- **Dependent error**
- **Bootstrapping**

# What Do I Consider a Simulation?

- **Often we think of big scary, hairy simulations with lots of parameters to vary, complex error structures, lots of complex formulas and always done by a biostatistician**
- **I consider everything from**
  - Demonstration of a concept
  - Creation of a static toy dataset with no randomness
  - Creation of a dataset based on probabilities
  - Varying parameters
  - Simulating error, and error structures
  - Big hairy simulations with lots of variation

# Simple Simulations

# Simulate an Exact Dataset

- data summary;
  - input **exp out** count;
  - cards;
  - 1 1 25
  - 1 0 75
  - 0 1 50
  - 0 0 50
  - ;
- run;
- proc freq data=summary;
  - tables exp\*dis/nocol noperc;
  - **weight count;**
- run;

Table of exp by dis

exp	dis		Total
Frequency	0	1	
Row Pct			
0	50 50.00	50 50.00	100
1	75 75.00	25 25.00	100
<b>Total</b>	<b>125</b>	<b>75</b>	<b>200</b>

The screenshot shows the SAS interface. The main window displays a table with the following data:

	exp	out	count
1	1	1	25
2	1	0	75
3	0	1	50
4	0	0	50

The Explorer window on the left shows the contents of the 'Work' session, including files named Conf, First, Second, and Third.

The taskbar at the bottom shows several open windows: Re..., Expl..., Output - (...), Log - (Un..., Sims for t..., and VIEWTA...



# Simulate an Exact Individual Level Dataset

	E+	E-
D+	25	50
D-	75	50
Total	100	100

- Create the 2x2 table
- data individual;
  - do j = 1 to 25;
    - exp = 1; dis = 1; output;
  - end;
  - do j = 1 to 75;
    - exp = 1; dis = 0; output;
  - end;
  - do j = 1 to 50;
    - exp = 0; dis = 1; output;
  - end;
  - do j = 1 to 50;
    - exp = 0; dis = 0; output;
  - end;
- run;

SAS - [VIEWTABLE: Work.Second]

	Exp	dis
20	1	1
21	1	1
22	1	1
23	1	1
24	1	1
25	1	1
26	1	0
27	1	0
28	1	0
29	1	0
30	1	0
31	1	0
32	1	0
33	1	0
34	1	0
35	1	0
36	1	0
37	1	0
38	1	0
39	1	0
40	1	0
41	1	0
42	1	0
43	1	0
44	1	0

Output - ... Log - (U... Sims for... VIEWT...

C:\Users\mfox



# Random Number Generators

- Often want to draw randomly from a distribution rather than create exact outputs
- **SAS** has lots of random number generators
  - `RAND('BERNOULLI', probability);`
  - `RANBIN(seed, # trials, probability);`
  - `RANUNI(seed);`
  - `RANTRI(seed, mode)`
  - `RANNOR(seed, x);`
  - and more... see SAS documentation



# Simulate a Simple Dataset Probabilistically

- $\Pr(E+)$  is 50%
- $\Pr(D+)$  is 25% if E-
- $\Pr(D+)$  is 50% if E+
  
- data prob;
  - do j = 1 to 10000;
    - exp = rand('bernoulli',0.5);
    - if exp = 0 then dis = rand('bernoulli',0.25);
    - else if exp = 1 then dis = rand('bernoulli',0.5);
    - output;
  - end;
- run;

exp	Frequency	Percent	Cumulative Frequency	Cumulative Percent
0	4985	49.85	4985	49.85
1	5015	50.15	10000	100.00

dis	Frequency	Percent	Cumulative Frequency	Cumulative Percent
0	6328	63.28	6328	63.28
1	3672	36.72	10000	100.00

Table of exp by dis

exp	dis		Total
Frequency Row Pct	0	1	
0	3759 75.41	1226 24.59	4985
1	2569 51.23	2446 48.77	5015
Total	6328	3672	10000

SAS - [VIEWTABLE: Work.Third]

	Exp	dis
1	0	0
2	0	1
3	1	1
4	1	1
5	0	0
6	1	1
7	0	0
8	1	0
9	1	1
10	0	0
11	1	0
12	1	1
13	1	1
14	0	0
15	0	0
16	0	0
17	0	1
18	0	0
19	1	1
20	0	1
21	1	0
22	1	0
23	0	0
24	1	0
25	0	0

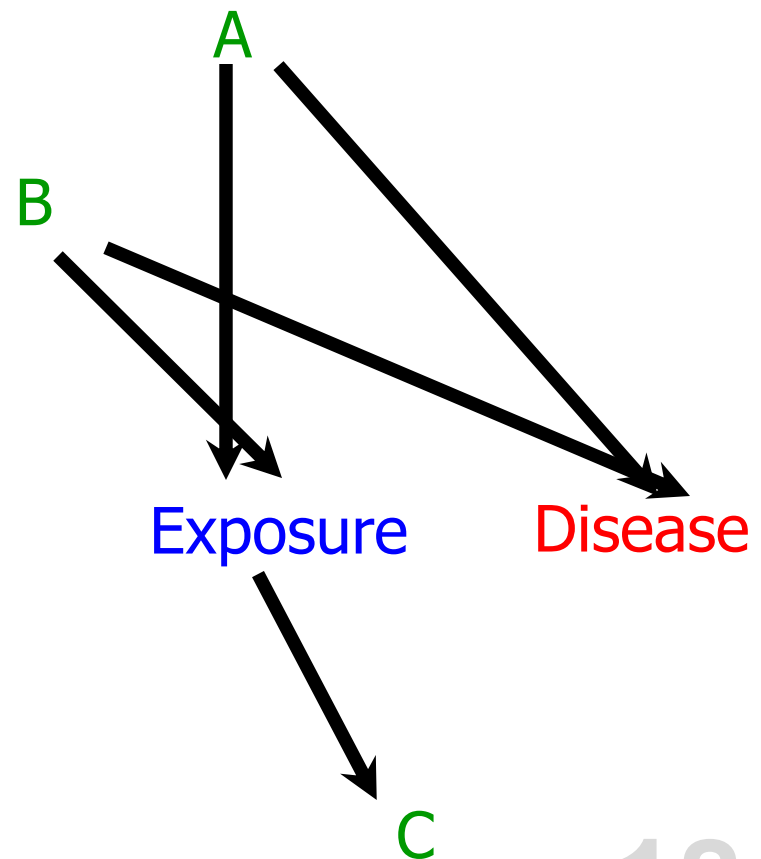
Output -... Log - (U... Sims for... VIEWTABLE...

C:\Users\mfox



# DAGs to Simulate Data

- There are other ways, for me this is the simplest
- Can simulate from a regression model
- (See book for details)
- Can build complex error structures



# Confounding

# N=1000 per stratum

## C should be associated with E and D

Crude			C-			C+		
	E+	E-		E+	E-		E+	E-
D+	160	170	D+	80	160	D+	80	10
D-	840	830	D-	120	640	D-	720	190
Total	1000	1000	Total	200	800	Total	800	200
Risk	0.16	0.17	Risk	0.4	0.2	Risk	0.1	0.05
RR	0.94		RR	2		RR	2	

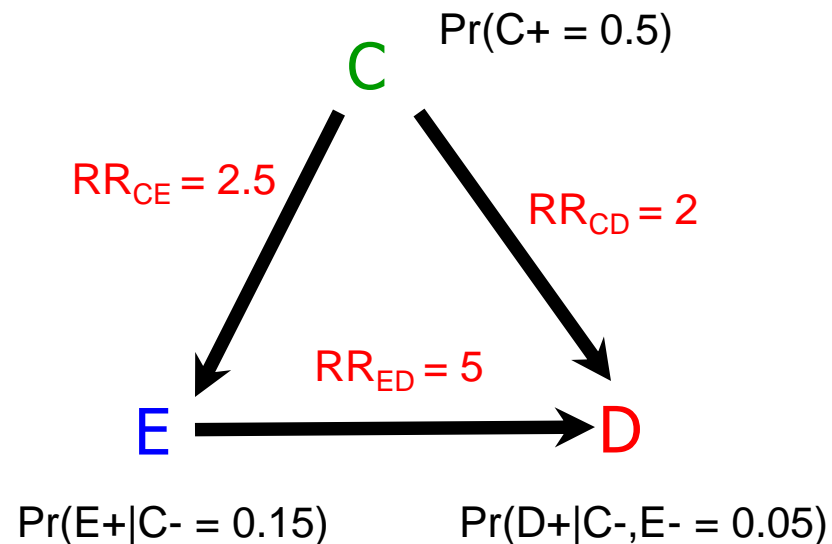
$$RR_{CD|E-} = 4 = (0.2/0.05)$$

$$RR_{CE} = 4 = [(800/1000)/(200/1000)]$$



# Simulating DAGs: Confounding

- **Define the baseline risks**
  - What % of people have C+?
  - What % of people C- are E+
  - What % of people C- and E- are D+
- **Define effects (relative vs absolute)**
  - What is the RR/RD for C on E?
  - What is the RR/RD for C on D?
  - What is the RR/RD for E on D?
- **Define interactions**
  - Do E and C interact to cause D?
  - If so, on what scale?



# Simulate Confounding Probabilistically

- data conf:
- di

Table of exp by dis

exp	dis		Total
	+	-	
+	1068 41.52	1504 58.48	2572
-	511 6.88	6917 93.12	7428
Total	1579	8421	10000

Statistics for Table of exp by dis

Estimates of the Relative Risk (Row1/Row2)

Type of Study	Value	95% Confidence Limits	
Case-Control (Odds Ratio)	9.6121	8.5315	10.8297
Cohort (Coll Risk)	<u>6.0360</u>	5.4867	6.6403

Estimates of the Common Relative Risk (Row1/Row2)

Type of Study	Method	Value	95% Confidence Limits	
Case-Control (Odds Ratio)	Mantel-Haenszel	8.2128	7.2595	9.2913
	Logit	7.9946	7.0651	9.0463
Cohort (Coll Risk)	Mantel-Haenszel	5.0930	4.6082	5.6289
	Logit	<u>5.0643</u>	4.5876	5.5906

- run;



SAS - (VIEWTABLE: Work.Conf)

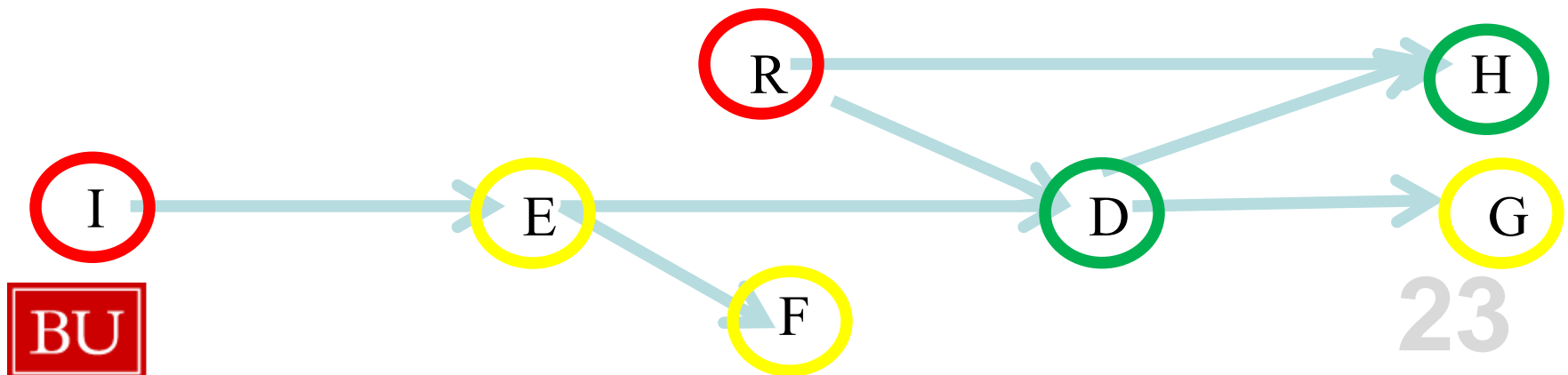
help

dis	
1	0
0	1
0	0
1	1
0	0
1	0
0	0
0	1
0	0
0	0
0	0
0	0
0	0
0	0
1	1
0	0
1	0
1	0
0	0
0	0
1	0
1	1
0	0
0	0

ns for... VIEWT...

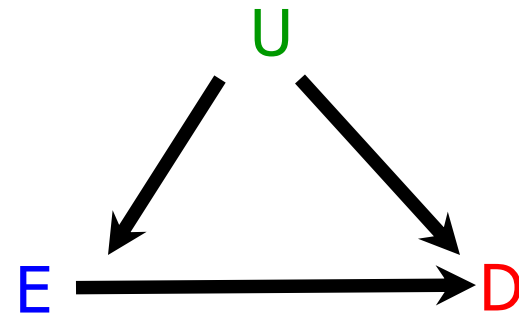
# Simulating DAGs

- Find the independent nodes and simulate
  - Specify probability
- Simulate nodes dependent on one arrow
  - Specify probability in all levels of the arrows the leads into the node
- Simulate nodes dependent on only two arrows, etc.
  - Specify probability in all levels of arrows that lead into the node
- Pay attention to scale, additive or multiplicative
- Pay attention to interaction (additive or multiplicative)



# Unmeasured Confounders

- Suppose I have data on E and D and want to simulate U?
- Now the E and D variables exist, can't simulate E and D dependent on U and C
- Instead I need to simulate U based on the probability of being in any of the 8 missing cells in the table
  - $RR_{UD} = 2.5$ ,  $\Pr(U+|E+) = 10\%$   $\Pr(U+|E-) = 20\%$



	Crude			U+			U-	
	E+	E-		E+	E-		E+	E-
D+	a 45	b 70	D+	A1	B1	D+	A0	B0
D-	c 255	d 630	D-	C1	D1	D-	C0	D0
Total	m 300	n 700	Total	M1 30	N1 140	Total	M0 270	N0 560



# Unmeasured Confounders

- $RR_{CD} = 2.5$  and
 
$$A_1 = \frac{RR_{CD}M_1a}{RR_{CD}M_1 + m - M_1} \quad A_1 = \frac{2.5 * 30 * 45}{2.5 * 30 + 300 - 30}$$

$$B_1 = \frac{RR_{CD}N_1b}{RR_{CD}N_1 + n - N_1} \quad B_1 = \frac{2.5 * 140 * 70}{2.5 * 140 + 700 - 140}$$
- So  $A_1 = 9.8$  and  $B_1 = 26.9$
- And we can now fill in the rest of the table

	Crude		U+			U-		
	E+	E-		E+	E-		E+	E-
D+	a 45	b 70	D+ <td>A<sub>1</sub> 9.8</td> <td>B<sub>1</sub> 26.9</td> <td>D+ <td>A<sub>0</sub> 35.2</td> <td>B<sub>0</sub> 43.1</td> </td>	A <sub>1</sub> 9.8	B <sub>1</sub> 26.9	D+ <td>A<sub>0</sub> 35.2</td> <td>B<sub>0</sub> 43.1</td>	A <sub>0</sub> 35.2	B <sub>0</sub> 43.1
D-	c 255	d 630	D- <td>C<sub>1</sub> 20.2</td> <td>D<sub>1</sub> 113.1</td> <td>D- <td>C<sub>0</sub> 234.8</td> <td>D<sub>0</sub> 526.9</td> </td>	C <sub>1</sub> 20.2	D <sub>1</sub> 113.1	D- <td>C<sub>0</sub> 234.8</td> <td>D<sub>0</sub> 526.9</td>	C <sub>0</sub> 234.8	D <sub>0</sub> 526.9
Total	m 300	n 700	Total	M <sub>1</sub> 30	N <sub>1</sub> 140	Total	M <sub>0</sub> 270	N <sub>0</sub> 560

# Unmeasured Confounders

- So now for any person, if I know their E and D I can tell you the probability of having U:
  - $\Pr(U+|E+,D+) = 9.8/45$ ,  $\Pr(U+|E+,D-) = 20.2/255$
  - $\Pr(U+|E-,D+) = 26.9/70$ ,  $\Pr(U+|E-,D-) = 113.1/630$
- Code:
  - if  $E=1$  and  $D=1$  then  $U = \text{rand}(\text{'bernoulli'}, 9.8/45)$ ;
  - else if  $E=1$  and  $D=0$  then  $U = \text{rand}(\text{'bernoulli'}, 20.2/255)$ ;
  - else if  $E=0$  and  $D=1$  then  $U = \text{rand}(\text{'bernoulli'}, 26.9/70)$ ;
  - else if  $E=0$  and  $D=0$  then  $U = \text{rand}(\text{'bernoulli'}, 113.1/630)$ ;

	Crude			U+			U-	
	E+	E-		E+	E-		E+	E-
D+	a 45	b 70	D+	A1 9.8	B1 26.9	D+	A0 35.2	B0 43.1
D-	c 255	d 630	D-	C1 20.2	D1 113.1	D-	C0 234.8	D0 526.9
Total	m 300	n 700	Total	M1 30	N1 140	Total	M0 270	N0 560

# Three Posters Here at SER

- **100-S Implications of Nondifferential Dependent Misclassification of Covariate and Exposure**
  - Kelly Getz and Alana Brennan
  - TUESDAY, JUNE 24, 2014 7-8:30 PM
- **112-S Understating the Relationship between Directed Acyclic Graphs (DAGs) and Data through Simulation Studies**
  - Julia Rohr
  - TUESDAY, JUNE 24, 2014
- **412-S When Does Adjustment for Predictors of Exposure Misclassification Increase Bias? A Simulation Study**
  - Samantha Parker and Mahsa Yazdy
  - WEDNESDAY, JUNE 25 5:00 – 6:30 pm

# Example: Dependent Error

- I had a student whom I asked to simulate dependent error to see when it mattered most
- A colleague had a student who wrote a paper on the same idea (Kelly Getz)
- We brought them together
- SimPLE was born

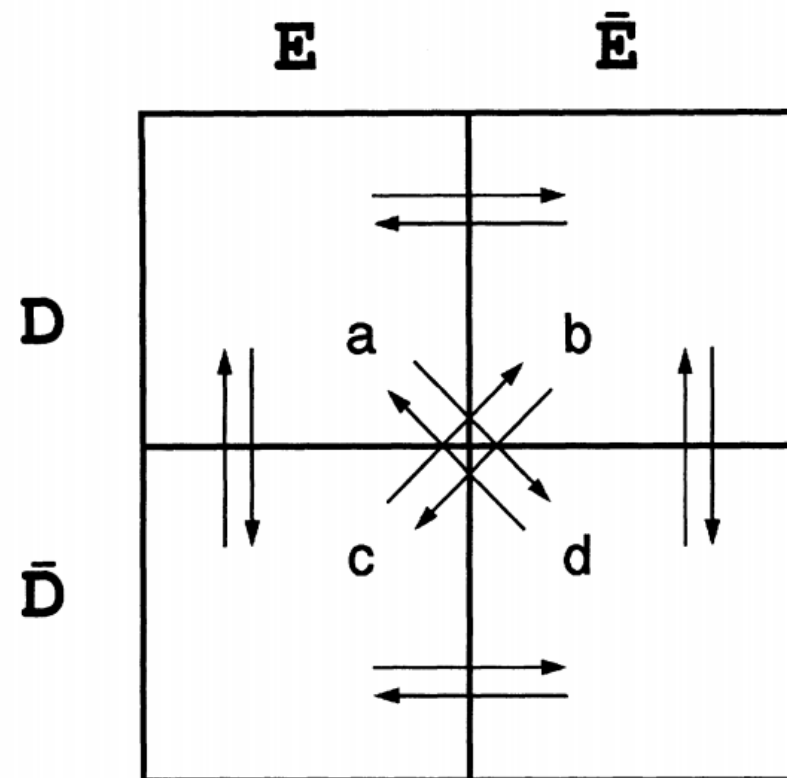


FIGURE 1. Rearrangement resulting from error in classification of exposure ( $E$ ,  $\bar{E}$ ) and outcome ( $D$ ,  $\bar{D}$ ).